

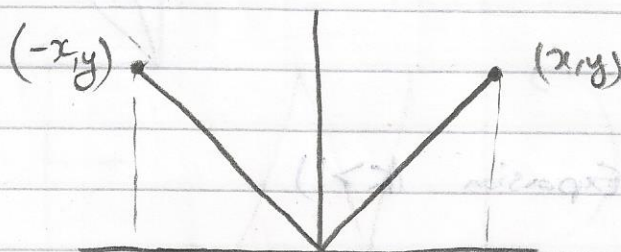
$$P^T A P = \begin{pmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The geometry of linear transformations in the plane

Reflection in the plane

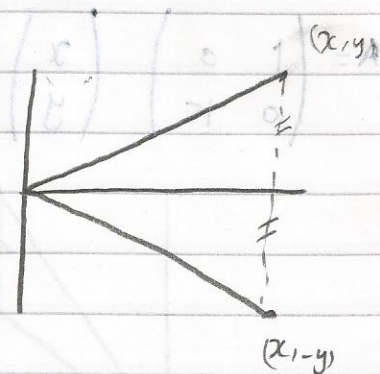
a) Reflection in the y-axis

$$A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{eg.} \quad \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ y \end{pmatrix}$$



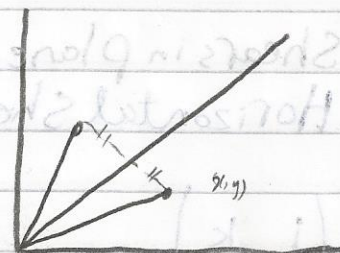
b) Reflection in the x-axis

$$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{eg.} \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$$



c) Reflection in the line $y=x$

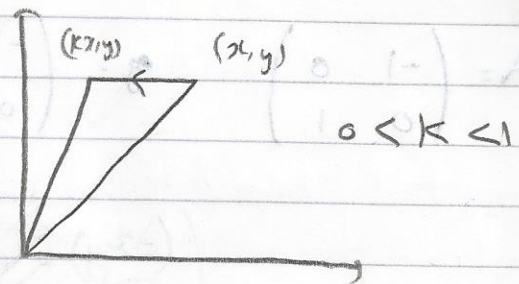
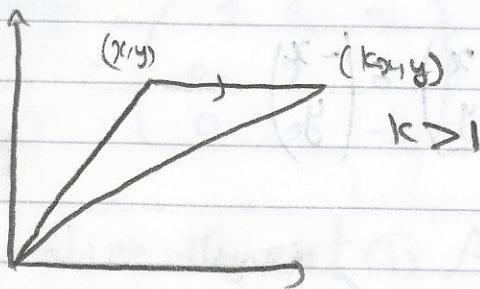
$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{eg.} \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$$



2- Expansions and contractions in the plane

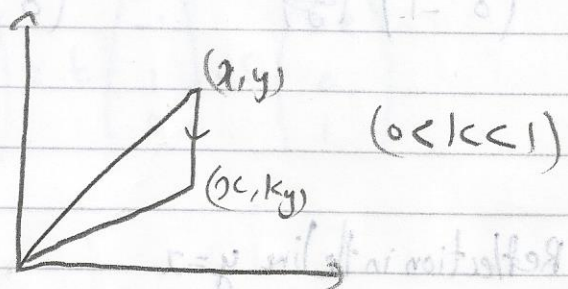
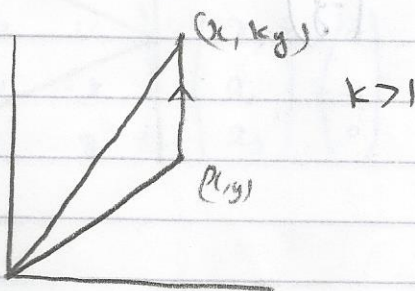
a) Horizontal $\begin{cases} \text{Expansion } (k > 1) \\ \text{contraction } (0 < k < 1) \end{cases}$

$$A = \begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} kx \\ y \end{pmatrix}$$



b) Vertical $\begin{cases} \text{Expansion } (k > 1) \\ \text{Contraction } (0 < k < 1) \end{cases}$

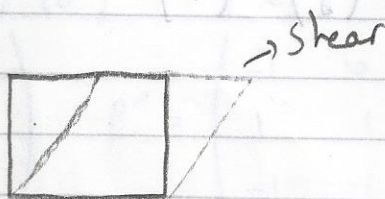
$$A = \begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ ky \end{pmatrix}$$

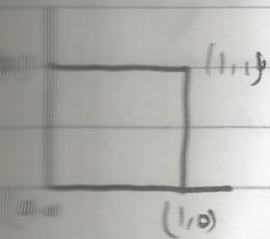


3- Shears in plane

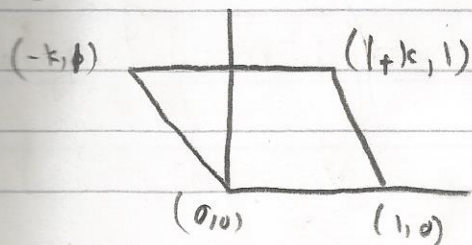
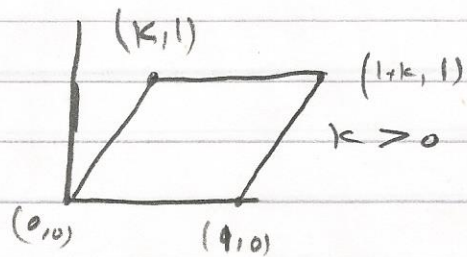
a) Horizontal Shear

$$A = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$$





$$A = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$$



$$k < 0$$

NOTES